Ch. 8 SYMMETRICAL COMPONENTS

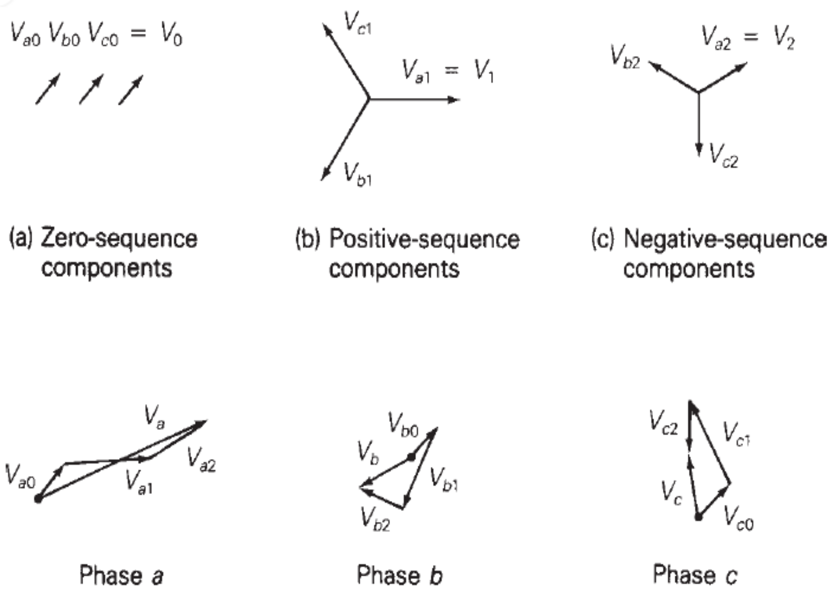
* It is a linear transformation from three-phase network into sequence network that contains symmetrical components.
* This conversion is important to decouple the three-phase network. the analysis of the faults in symmetrical network is easier as we will see in Ch. 9.

8.1 Definition of Symmetrical Components:

* The set of sequence components:

1. *Zero-sequence* components, consisting of three phasors with equal magnitudes and with zero phase displacement, as shown in Figure 8.1(a)
2. *Positive-sequence* components, consisting of three phasors with equal magnitudes, ±120° phase displacement, and positive sequence, as in Figure 8.1(b)
3. *Negative-sequence* components, consisting of three phasors with equal magnitudes, ±120° phase displacement, and negative sequence, as in Figure 8.1(c)

FIGURE 8.1
Resolving phase voltages into three sets of sequence components



The transformation equation:

$$\underbrace{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}_{V_P} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}}_{V_S} \Longrightarrow \begin{aligned} V_a &= V_0 + V_1 + V_2 \\ V_b &= V_0 + a^2 V_1 + a V_2 \\ V_c &= V_0 + a V_1 + a^2 V_2 \end{aligned}$$

$$V_P = A\, V_S$$

Where

$$a = 1\angle 120° = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$

The inverse of the A matrix:

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

TABLE 8.1

Common identities involving $a = 1\angle 120°$

$a^4 = a = 1\angle 120°$
$a^2 = 1\angle 240°$
$a^3 = 1\angle 0°$
$1 + a + a^2 = 0$
$1 - a = \sqrt{3}\angle -30°$
$1 - a^2 = \sqrt{3}\angle +30°$
$a^2 - a = \sqrt{3}\angle 270°$
$ja = 1\angle 210°$
$1 + a = -a^2 = 1\angle 60°$
$1 + a^2 = -a = 1\angle -60°$
$a + a^2 = -1 = 1\angle 180°$

$$V_s = A^{-1} V_p \Rightarrow \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

* It can also apply to the current:

$$\boxed{I_p = A\,I_s} \quad \& \quad \boxed{I_s = A^{-1} I_p}$$

* In Y-connected systems:

$$I_n = I_a + I_b + I_c = 3I_0$$

EXAMPLE 8.1 Sequence components: balanced line-to-neutral voltages

Calculate the sequence components of the following balanced line-to-neutral voltages with *abc* sequence:

$$V_p = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 277\angle 0° \\ 277\angle -120° \\ 277\angle +120° \end{bmatrix} \quad \text{volts}$$

SOLUTION Using (8.1.13)–(8.1.15):

$$V_0 = \tfrac{1}{3}[277\angle 0° + 277\angle -120° + 277\angle +120°] = 0$$

$$V_1 = \tfrac{1}{3}[277\angle 0° + 277\angle(-120° + 120°) + 277\angle(120° + 240°)]$$
$$= 277\angle 0° \quad \text{volts} = V_{an}$$

$$V_2 = \tfrac{1}{3}[277\angle 0° + 277\angle(-120° + 240°) + 277\angle(120° + 120°)]$$
$$= \tfrac{1}{3}[277\angle 0° + 277\angle 120° + 277\angle 240°] = 0$$

This example illustrates the fact that balanced three-phase systems with *abc* sequence (or positive sequence) have no zero-sequence or negative-sequence components. For this example, the positive-sequence voltage V_1 equals V_{an}, and the zero-sequence and negative-sequence voltages are both zero. ■

EXAMPLE 8.2 Sequence components: balanced *acb* currents

A Y-connected load has balanced currents with *acb* sequence given by

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 10\underline{/+120^\circ} \\ 10\underline{/-120^\circ} \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents.

SOLUTION Using (8.1.23)–(8.1.25):

$$I_0 = \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/120^\circ} + 10\underline{/-120^\circ}] = 0$$

$$\begin{aligned} I_1 &= \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/(120^\circ + 120^\circ)} + 10\underline{/(-120^\circ + 240^\circ)}] \\ &= \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/240^\circ} + 10\underline{/120^\circ}] = 0 \end{aligned}$$

$$\begin{aligned} I_2 &= \tfrac{1}{3}[10\underline{/0^\circ} + 10\underline{/(120^\circ + 240^\circ)} + 10\underline{/(-120^\circ + 120^\circ)}] \\ &= 10\underline{/0^\circ} \text{ A} = I_a \end{aligned}$$

This example illustrates the fact that balanced three-phase systems with *acb* sequence (or negative sequence) have no zero-sequence or positive-sequence components. For this example the negative-sequence current I_2 equals I_a, and the zero-sequence and positive-sequence currents are both zero. ■

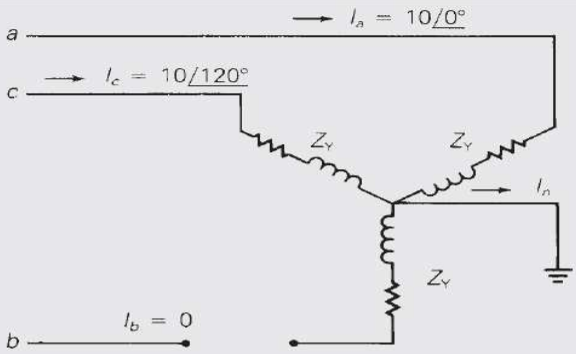
EXAMPLE 8.3 Sequence components: unbalanced currents

A three-phase line feeding a balanced-Y load has one of its phases (phase *b*) open. The load neutral is grounded, and the unbalanced line currents are

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 0 \\ 10\underline{/120^\circ} \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents and the neutral current.

FIGURE 8.2
Circuit for Example 8.3



SOLUTION The circuit is shown in Figure 8.2. Using (8.1.23)–(8.1.25):

$$\begin{aligned} I_0 &= \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/120^\circ}] \\ &= 3.333\underline{/60^\circ} \quad \text{A} \end{aligned}$$

$$I_1 = \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 240^\circ)}] = 6.667\underline{/0^\circ} \quad \text{A}$$

$$\begin{aligned} I_2 &= \tfrac{1}{3}[10\underline{/0^\circ} + 0 + 10\underline{/(120^\circ + 120^\circ)}] \\ &= 3.333\underline{/-60^\circ} \quad \text{A} \end{aligned}$$

Using (8.1.26) the neutral current is

$$\begin{aligned} I_n &= (10\underline{/0^\circ} + 0 + 10\underline{/120^\circ}) \\ &= 10\underline{/60^\circ} \text{ A} = 3I_0 \end{aligned}$$

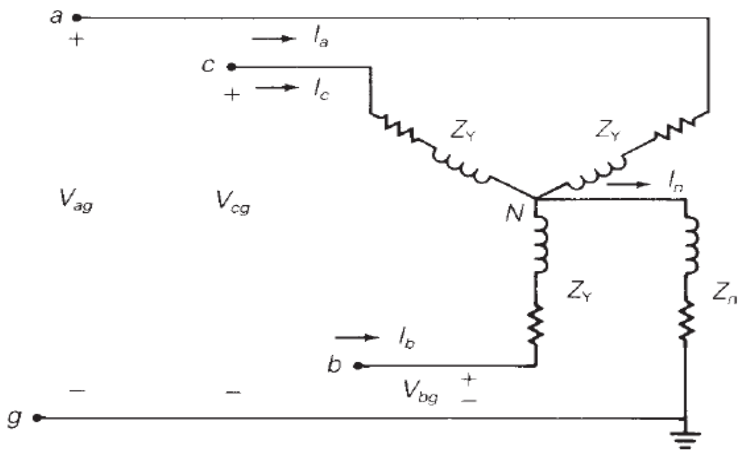
This example illustrates the fact that *unbalanced* three-phase systems may have nonzero values for all sequence components. Also, the neutral current equals three times the zero-sequence current, as given by (8.1.27). ■

8.2 Sequence Networks of Impedance Loads:

① Balanced Y-Connected Loads:

FIGURE 8.3

Balanced-Y impedance load



Using KVL:

$$\underbrace{\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}}_{V_p} = \underbrace{\begin{bmatrix} (Z_Y + Z_n) & Z_n & Z_n \\ Z_n & (Z_Y + Z_n) & Z_n \\ Z_n & Z_n & (Z_Y + Z_n) \end{bmatrix}}_{Z_p} \underbrace{\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}}_{I_p}$$

$$V_p = Z_p I_p$$

$$A V_s = Z_p A I_s \Rightarrow V_s = (A^{-1} Z_p A) I_s = V_s = Z_s I_s$$

Sequence Impedance matrix $\quad Z_s = A^{-1} Z_p A = \begin{bmatrix} (Z_Y + 3Z_n) & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$

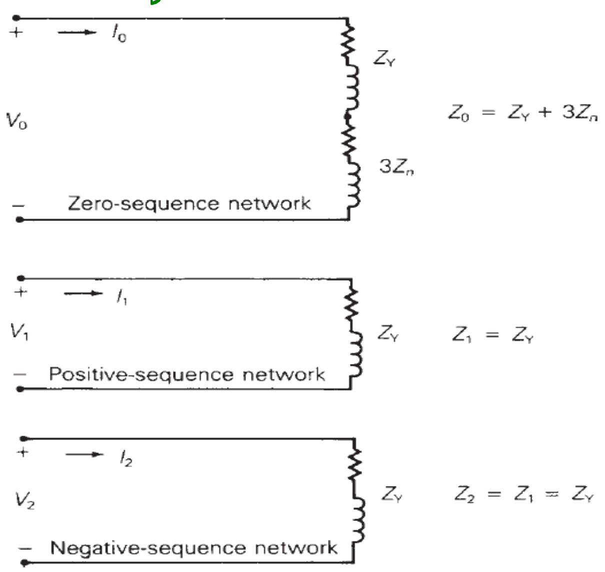
$V_0 = (Z_Y + 3Z_n) I_0 = Z_0 I_0$ zero-sequence impedance

$V_1 = Z_Y I_1 = Z_1 I_1$ positive-sequence impedance

$V_2 = Z_Y I_2 = Z_2 I_2$ negative-sequence impedance

FIGURE 8.4

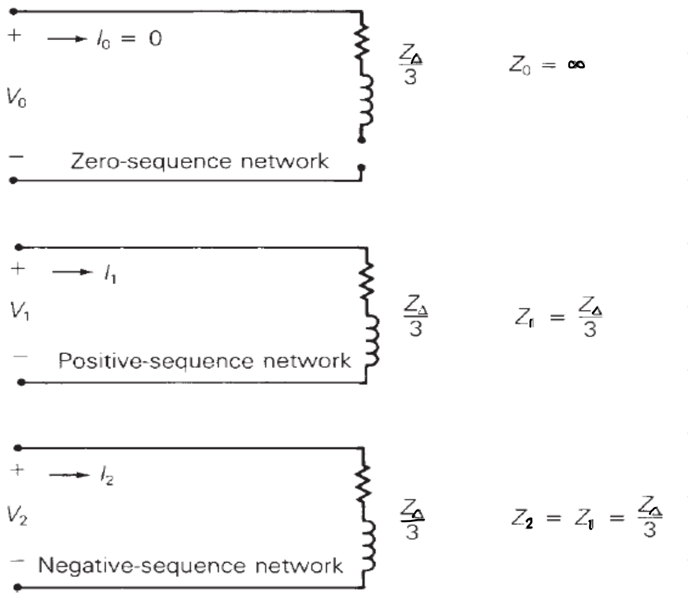
Sequence networks of a balanced-Y load



(2) Balanced Δ-Connected Loads:

FIGURE 8.5

Sequence networks for an equivalent Y representation of a balanced-Δ load



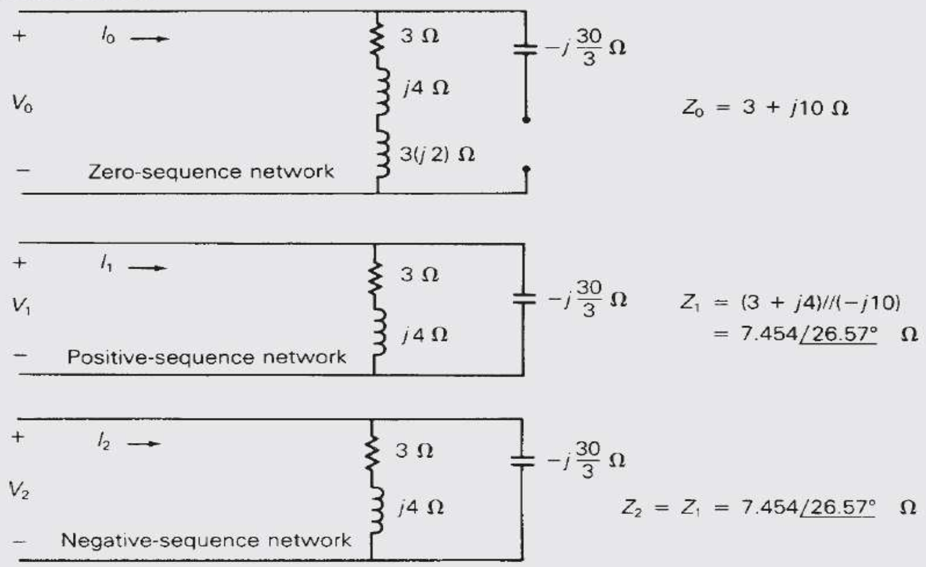
EXAMPLE 8.4 **Sequence networks: balanced-Y and balanced-Δ loads**

A balanced-Y load is in parallel with a balanced-Δ-connected capacitor bank. The Y load has an impedance $Z_Y = (3 + j4)\ \Omega$ per phase, and its neutral is grounded through an inductive reactance $X_n = 2\ \Omega$. The capacitor bank has a reactance $X_c = 30\ \Omega$ per phase. Draw the sequence networks for this load and calculate the load-sequence impedances.

SOLUTION The sequence networks are shown in Figure 8.6. As shown, the Y-load impedance in the zero-sequence network is in series with three times the neutral impedance. Also, the Δ-load branch in the zero-sequence network is open, since no zero-sequence current flows into the Δ load. In the positive- and negative-sequence circuits, the Δ-load impedance is divided by 3 and placed in parallel with the Y-load impedance. The equivalent sequence impedances are

FIGURE 8.6

Sequence networks for Example 8.4



$$Z_0 = Z_Y + 3Z_n = 3 + j4 + 3(j2) = 3 + j10 \quad \Omega$$

$$Z_1 = Z_Y // (Z_\Delta/3) = \frac{(3 + j4)(-j30/3)}{3 + j4 - j(30/3)}$$

$$= \frac{(5\underline{/53.13^\circ})(10\underline{/-90^\circ})}{6.708\underline{/-63.43^\circ}} = 7.454\underline{/26.57^\circ} \quad \Omega$$

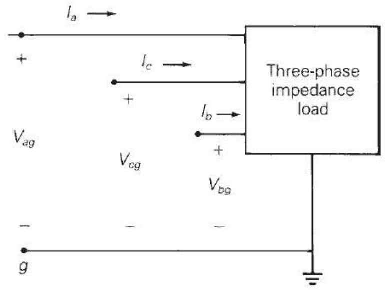
$$Z_2 = Z_1 = 7.454\underline{/26.57^\circ} \quad \Omega$$

■

③ General Impedance Loads:

FIGURE 8.7

General three-phase impedance load (linear, bilateral network, nonrotating equipment)



$$\begin{bmatrix} Z_0 & Z_{01} & Z_{02} \\ Z_{10} & Z_1 & Z_{12} \\ Z_{20} & Z_{21} & Z_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Diagonal sequence impedances

$$Z_0 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc})$$

$$Z_1 = Z_2 = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc})$$

Off-diagonal sequence impedances

$$Z_{01} = Z_{20} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc} - aZ_{ab} - a^2 Z_{ac} - Z_{bc})$$

$$Z_{02} = Z_{10} = \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc} - a^2 Z_{ab} - aZ_{ac} - Z_{bc})$$

$$Z_{12} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + aZ_{cc} + 2aZ_{ab} + 2a^2 Z_{ac} + 2Z_{bc})$$

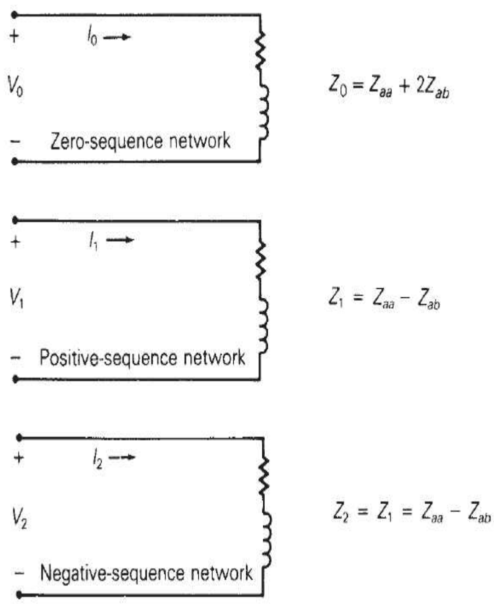
$$Z_{21} = \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2 Z_{cc} + 2a^2 Z_{ab} + 2aZ_{ac} + 2Z_{bc})$$

④ Symmetrical Loads:

$$\left.\begin{array}{l} (1)\ Z_{aa} = Z_{bb} = Z_{cc} \\ \text{and} \\ (2)\ Z_{ab} = Z_{ac} = Z_{bc} \end{array}\right\}\ \text{conditions for a symmetrical load}$$

FIGURE 8.8

Sequence networks of a three-phase symmetrical impedance load (linear, bilateral network, nonrotating equipment)



Then $Z_{01} = Z_{10} = Z_{02} = Z_{20} = Z_{12} = Z_{21} = 0$

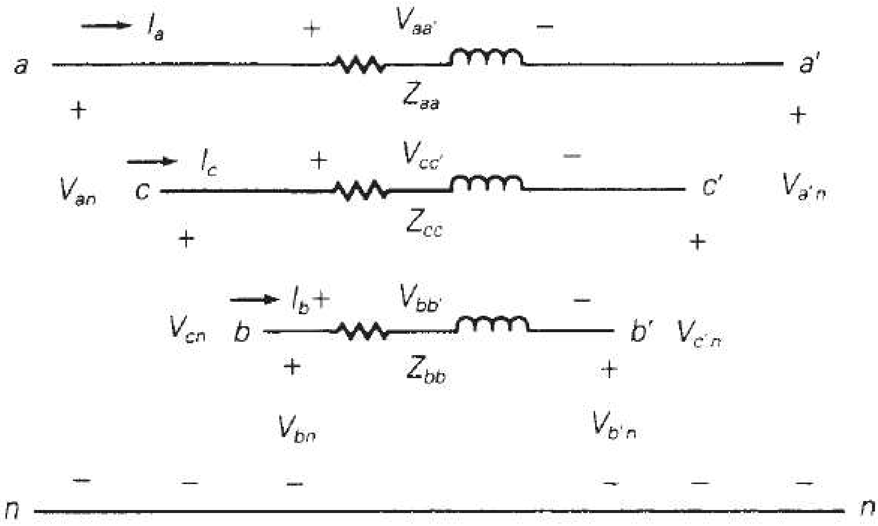
$$Z_0 = Z_{aa} + 2Z_{ab}$$

$$Z_1 = Z_2 = Z_{aa} - Z_{ab}$$

8.3 Sequence Networks of Series Impedance: (Transmission Lines)

FIGURE 8.9

Three-phase series impedances (linear, bilateral network, nonrotating equipment)



Using KVL:

$$\begin{bmatrix} V_{an} - V_{a'n} \\ V_{bn} - V_{b'n} \\ V_{cn} - V_{c'n} \end{bmatrix} = \begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$V_p - V_{p'} = Z_p I_p$$

$$V_s - V_{s'} = Z_s I_s$$

Where $Z_s = A^{-1} Z_p A$

When

① $Z_{aa} = Z_{bb} = Z_{cc}$

and

② $Z_{ab} = Z_{ac} = Z_{bc}$

conditions for symmetrical series impedances

Then

$$Z_s = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

where

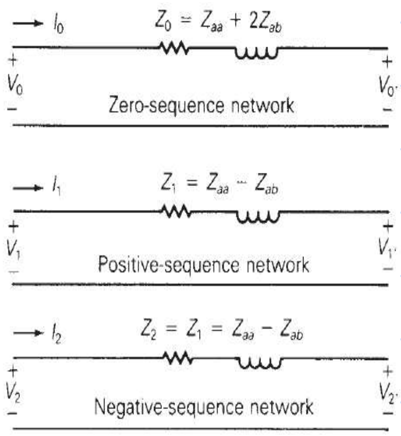
$$Z_0 = Z_{aa} + 2Z_{ab}$$

and

$$Z_1 = Z_2 = Z_{aa} - Z_{ab}$$

FIGURE 8.10

Sequence networks of three-phase symmetrical series impedances (linear, bilateral network, nonrotating equipment)

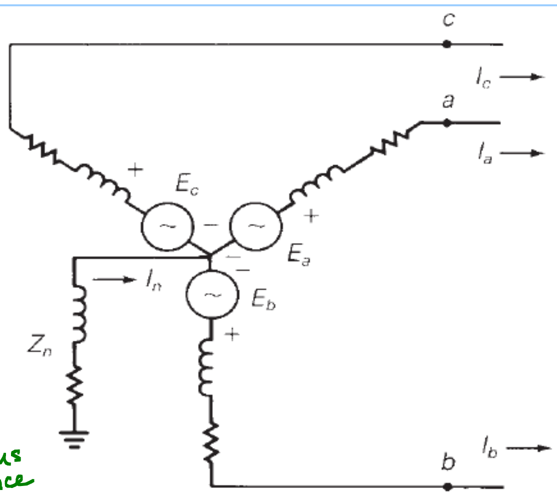


8.5 Sequence Networks of Rotating Machines:

(A) Synchronous Generators:

FIGURE 8.13

Y-connected synchronous generator



synchronous impedance

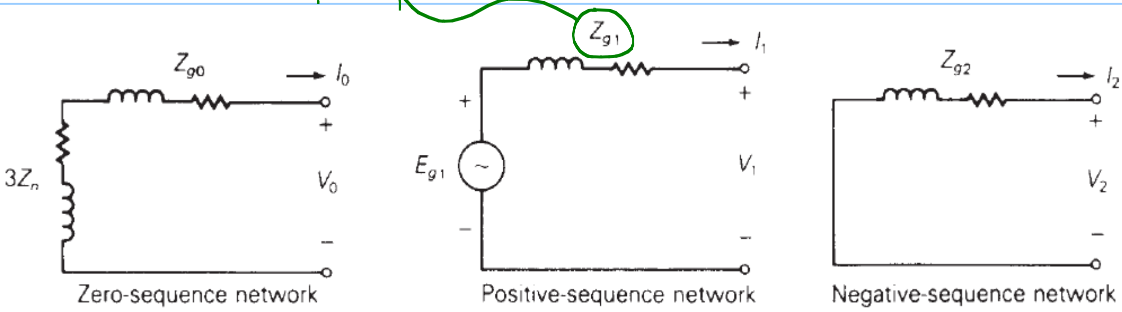


FIGURE 8.14 Sequence networks of a Y-connected synchronous generator

(B) Synchronous & Induction Motors:

FIGURE 8.15

Sequence networks of three-phase motors

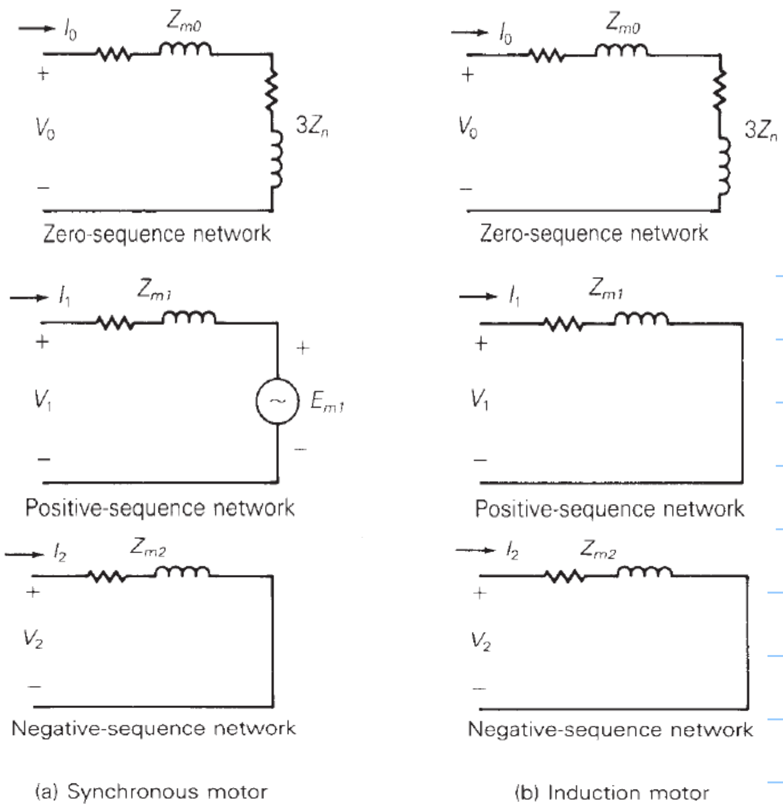


TABLE A.1

Typical average values of synchronous-machine constants

Constant (units)	Type	Symbol	Turbo-Generator (solid rotor)	Water-Wheel Generator (with dampers)	Synchro-nous Condenser	Synchro-nous Motor
Reactances (per unit)	Synchronous	X_d	1.1	1.15	1.80	1.20
		X_q	1.08	0.75	1.15	0.90
	Transient	X_d'	0.23	0.37	0.40	0.35
		X_q'	0.23	0.75	1.15	0.90
	Subtransient	X_d''	0.12	0.24	0.25	0.30
		X_q''	0.15	0.34	0.30	0.40
	Negative-sequence	X_2	0.13	0.29	0.27	0.35
	Zero-sequence	X_0	0.05	0.11	0.09	0.16
Resistances (per unit)	Positive-sequence	R (dc)	0.003	0.012	0.008	0.01
		R (ac)	0.005	0.012	0.008	0.01
	Negative-sequence	R_2	0.035	0.10	0.05	0.06
Time constants (seconds)	Transient	T_{d0}'	5.6	5.6	9.0	6.0
		T_d'	1.1	1.8	2.0	1.4
	Subtransient	$T_d'' = T_q''$	0.035	0.035	0.035	0.036
	Armature	T_a	0.16	0.15	0.17	0.15

(Adapted from E. W. Kimbark, *Power System Stability: Synchronous Machines* (New York: Dover Publications, 1956/1968), Chap. 12)

EXAMPLE 8.5 **Currents in sequence networks**

Draw the sequence networks for the circuit of Example 2.5 and calculate the sequence components of the line current. Assume that the generator neutral is grounded through an impedance $Z_n = j10\ \Omega$, and that the generator sequence impedances are $Z_{g0} = j1\ \Omega$, $Z_{g1} = j15\ \Omega$, and $Z_{g2} = j3\ \Omega$.

SOLUTION The sequence networks are shown in Figure 8.16. They are obtained by interconnecting the sequence networks for a balanced-Δ load, for

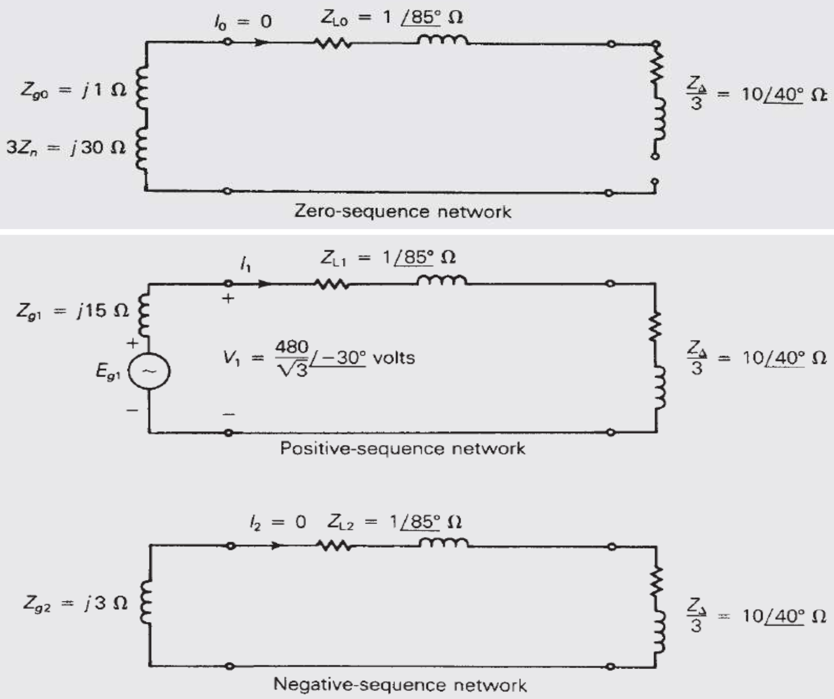


FIGURE 8.16 Sequence networks for Example 8.5

series-line impedances, and for a synchronous generator, which are given in Figures 8.5, 8.10, and 8.14.

It is clear from Figure 8.16 that $I_0 = I_2 = 0$ since there are no sources in the zero- and negative-sequence networks. Also, the positive-sequence generator terminal voltage V_1 equals the generator line-to-neutral terminal voltage. Therefore, from the positive-sequence network shown in the figure and from the results of Example 2.5,

$$I_1 = \frac{V_1}{(Z_{L1} + \frac{1}{3}Z_\Delta)} = 25.83\underline{/-73.78°}\ \text{A} = I_a$$

Note that from (8.1.20), I_1 equals the line current I_a, since $I_0 = I_2 = 0$. ■

EXAMPLE 8.6 **Solving unbalanced three-phase networks using sequence components**

A Y-connected voltage source with the following unbalanced voltage is applied to the balanced line and load of Example 2.5.

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 260\underline{/-120^\circ} \\ 295\underline{/+115^\circ} \end{bmatrix} \quad \text{volts}$$

The source neutral is solidly grounded. Using the method of symmetrical components, calculate the source currents I_a, I_b, and I_c.

SOLUTION Using (8.1.13)–(8.1.15), the sequence components of the source voltages are:

$$\begin{aligned} V_0 &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ} + 295\underline{/115^\circ}) \\ &= 7.4425 + j14.065 = 15.912\underline{/62.11^\circ} \quad \text{volts} \\ V_1 &= \tfrac{1}{3}(227\underline{/0^\circ} + 260\underline{/-120^\circ + 120^\circ} + 295\underline{/115^\circ + 240^\circ}) \\ &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/0^\circ} + 295\underline{/-5^\circ}) \\ &= 276.96 - j8.5703 = 277.1\underline{/-1.772^\circ} \quad \text{volts} \\ V_2 &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/-120^\circ + 240^\circ} + 295\underline{/115^\circ + 120^\circ}) \\ &= \tfrac{1}{3}(277\underline{/0^\circ} + 260\underline{/120^\circ} + 295\underline{/235^\circ}) \\ &= -7.4017 - j5.4944 = 9.218\underline{/216.59^\circ} \quad \text{volts} \end{aligned}$$

These sequence voltages are applied to the sequence networks of the line and load, as shown in Figure 8.17. The sequence networks of this figure

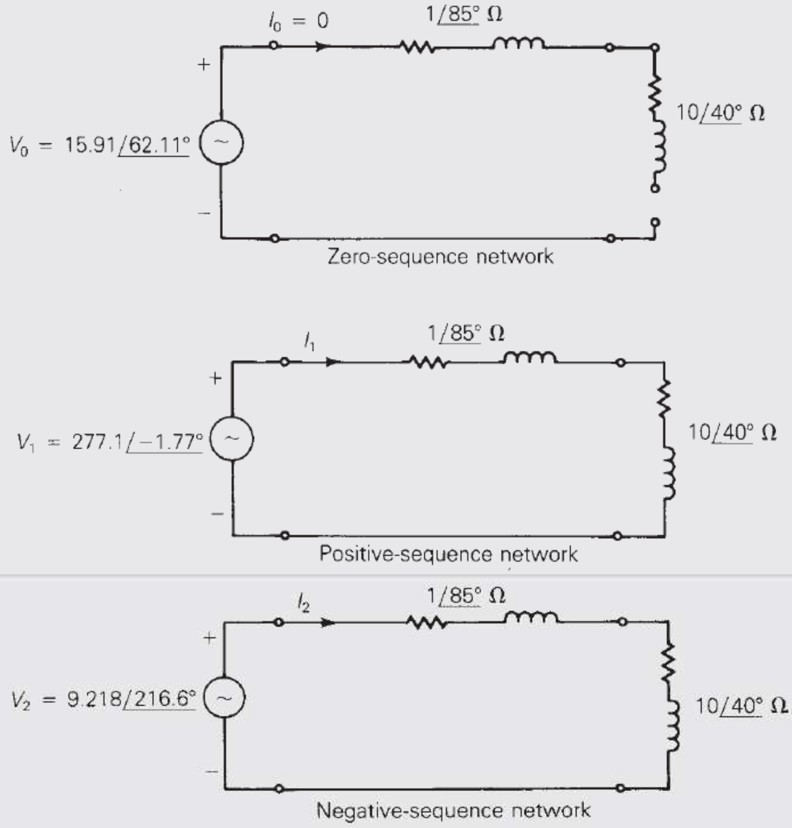


FIGURE 8.17 Sequence networks for Example 8.6

are uncoupled, and the sequence components of the source currents are easily calculated as follows:

$$I_0 = 0$$

$$I_1 = \frac{V_1}{Z_{L1} + \dfrac{Z_\Delta}{3}} = \frac{277.1\underline{/-1.772^\circ}}{10.73\underline{/43.78^\circ}} = 25.82\underline{/-45.55^\circ} \quad \text{A}$$

$$I_2 = \frac{V_2}{Z_{L2} + \frac{Z_\Delta}{3}} = \frac{9.218\underline{/216.59°}}{10.73\underline{/43.78°}} = 0.8591\underline{/172.81°} \quad \text{A}$$

Using (8.1.20)–(8.1.22), the source currents are:

$$I_a = (0 + 25.82\underline{/-45.55°} + 0.8591\underline{/172.81°})$$
$$= 17.23 - j18.32 = 25.15\underline{/-46.76°} \quad \text{A}$$
$$I_b = (0 + 25.82\underline{/-45.55° + 240°} + 0.8591\underline{/172.81° + 120°})$$
$$= (25.82\underline{/194.45°} + 0.8591\underline{/292.81°})$$
$$= -24.67 - j7.235 = 25.71\underline{/196.34°} \quad \text{A}$$
$$I_c = (0 + 25.82\underline{/-45.55° + 120°} + 0.8591\underline{/172.81° + 240°})$$
$$= (25.82\underline{/74.45°} + 0.8591\underline{/52.81°})$$
$$= 7.441 + j25.56 = 26.62\underline{/73.77°} \quad \text{A}$$

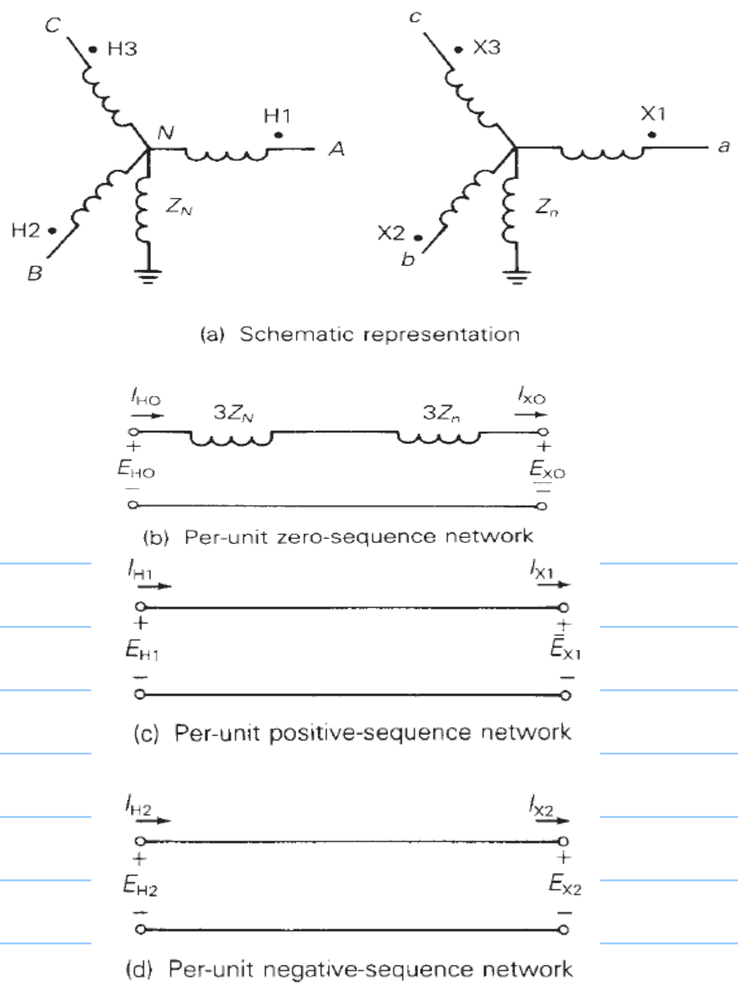
You should calculate the line currents for this example without using symmetrical components, in order to verify this result and to compare the two solution methods (see Problem 8.33). Without symmetrical components, coupled KVL equations must be solved. With symmetrical components, the conversion from phase to sequence components decouples the networks as well as the resulting KVL equations, as shown above. ■

8.6 Per-Unit Sequence Models of Three-Phase Two-Winding Transformers:

FIGURE 8.18

Ideal Y–Y transformer

Balanced



(a) Schematic representation

(b) Per-unit zero-sequence network

(c) Per-unit positive-sequence network

(d) Per-unit negative-sequence network

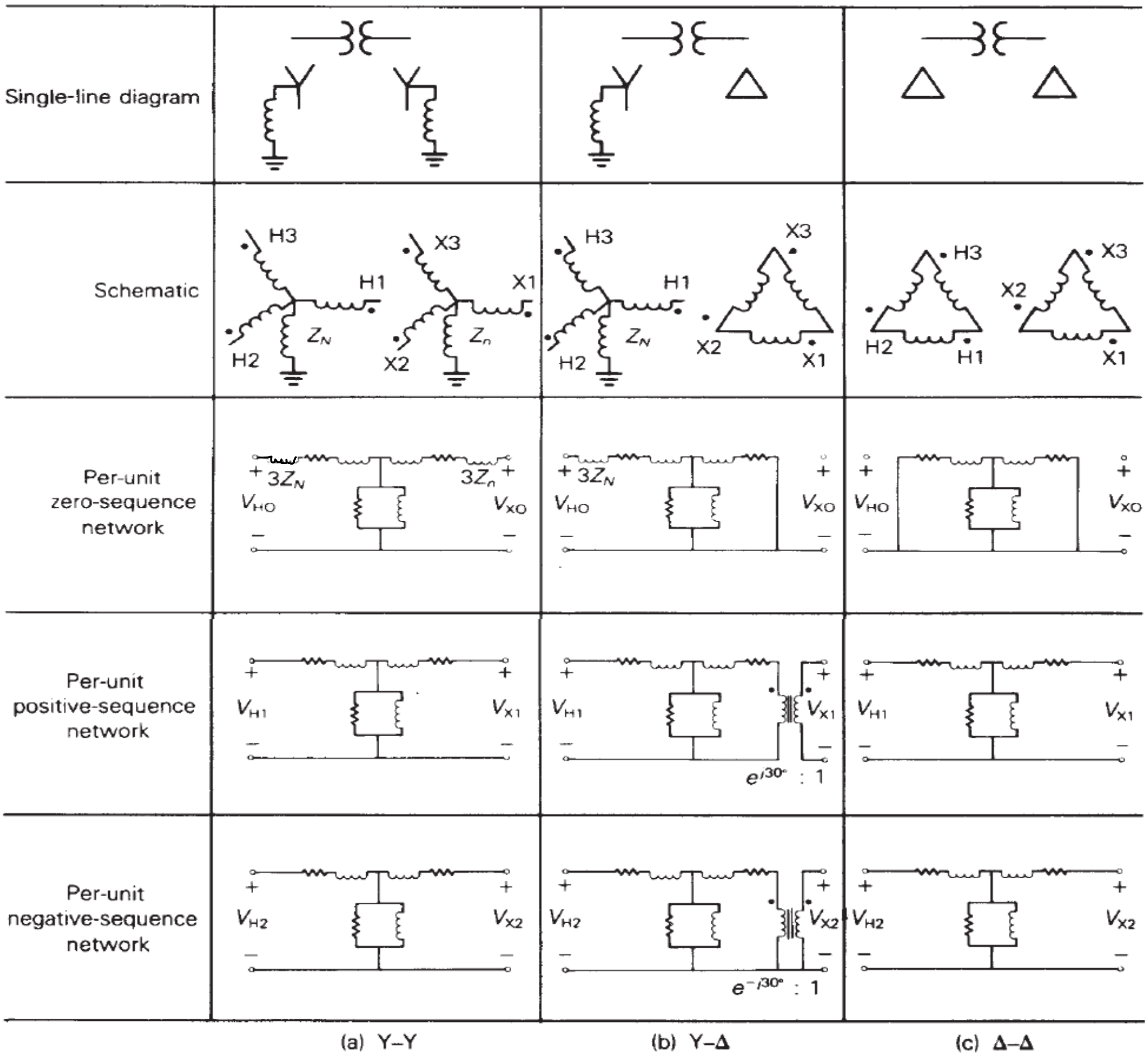


FIGURE 8.19 Per-unit sequence networks of practical Y–Y, Y–Δ, and Δ–Δ transformers

* Y-Δ transformer features:

1. The per-unit impedances do not depend on the winding connections. That is, the per-unit impedances of a transformer that is connected Y–Y, Y–Δ, Δ–Y, or Δ–Δ are the same. However, the base voltages do depend on the winding connections.

2. A phase shift is included in the per-unit positive- and negative-sequence networks. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30°. For negative sequence, the high-voltage quantities lag by 30°.

3. Zero-sequence currents can flow in the Y winding if there is a neutral connection, and corresponding zero-sequence currents flow within the Δ winding. However, no zero-sequence current enters or leaves the Δ winding.

* Δ-Δ transformer features:

1. The positive- and negative-sequence networks, which are identical, are the same as those for the Y–Y transformer. It is assumed that the windings are labeled so there is no phase shift. Also, the per-unit impedances do not depend on the winding connections, but the base voltages do.

2. Zero-sequence currents *cannot* enter or leave either Δ winding, although they can circulate within the Δ windings.

EXAMPLE 8.7 Solving unbalanced three-phase networks with transformers using per-unit sequence components

A 75-kVA, 480-volt Δ/208-volt Y transformer with a solidly grounded neutral is connected between the source and line of Example 8.6. The transformer leakage reactance is $X_{eq} = 0.10$ per unit; winding resistances and exciting current are neglected. Using the transformer ratings as base quantities, draw the per-unit sequence networks and calculate the phase a source current I_a.

SOLUTION The base quantities are $S_{base1\phi} = 75/3 = 25$ kVA, $V_{baseHLN} = 480/\sqrt{3} = 277.1$ volts, $V_{baseXLN} = 208/\sqrt{3} = 120.1$ volts, and $Z_{baseX} = (120.1)^2/25{,}000 = 0.5770\ \Omega$. The sequence components of the actual source voltages are given in Figure 8.17. In per-unit, these voltages are

$$V_0 = \frac{15.91\underline{/62.11^\circ}}{277.1} = 0.05742\underline{/62.11^\circ} \quad \text{per unit}$$

$$V_1 = \frac{277.1\underline{/-1.772^\circ}}{277.1} = 1.0\underline{/-1.772^\circ} \quad \text{per unit}$$

$$V_2 = \frac{9.218\underline{/216.59^\circ}}{277.1} = 0.03327\underline{/216.59^\circ} \quad \text{per unit}$$

The per-unit line and load impedances, which are located on the low-voltage side of the transformer, are

$$Z_{L0} = Z_{L1} = Z_{L2} = \frac{1\underline{/85^\circ}}{0.577} = 1.733\underline{/85^\circ} \quad \text{per unit}$$

$$Z_{load1} = Z_{load2} = \frac{Z_\Delta}{3(0.577)} = \frac{10\underline{/40^\circ}}{0.577} = 17.33\underline{/40^\circ} \quad \text{per unit}$$

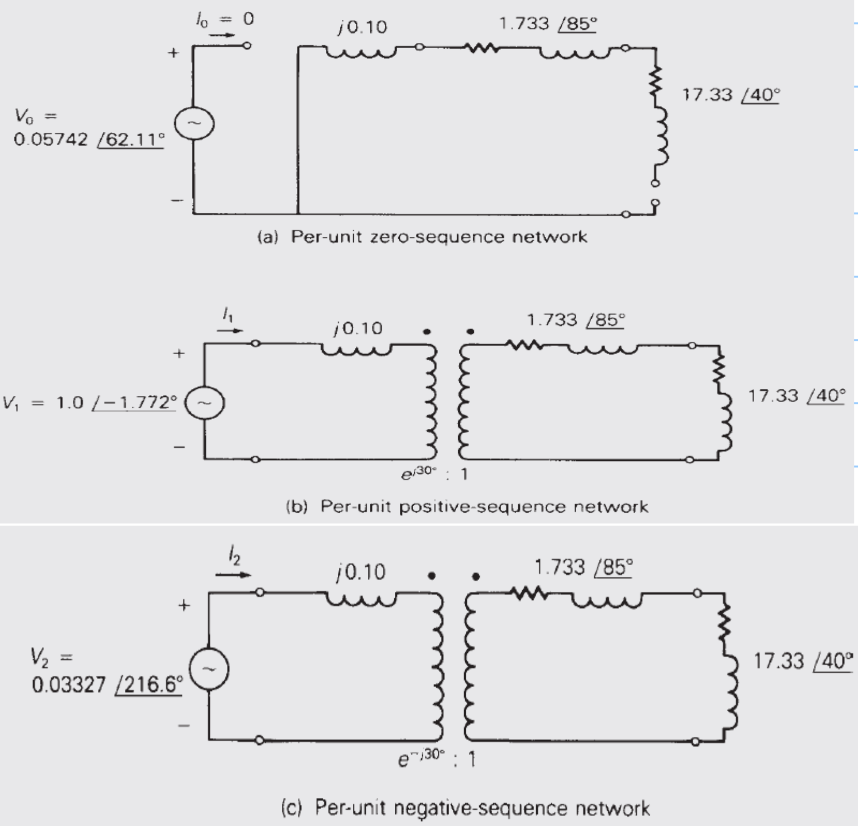


FIGURE 8.20 Per-unit sequence networks for Example 8.7

The per-unit sequence networks are shown in Figure 8.20. Note that the per-unit line and load impedances, when referred to the high-voltage side of the phase-shifting transformer, do not change [(see (3.1.26)]. Therefore, from Figure 8.20, the sequence components of the source currents are

$$I_0 = 0$$

$$I_1 = \frac{V_1}{jX_{eq} + Z_{L1} + Z_{load1}} = \frac{1.0\underline{/-1.772^\circ}}{j0.10 + 1.733\underline{/85^\circ} + 17.33\underline{/40^\circ}}$$

$$= \frac{1.0\underline{/-1.772^\circ}}{13.43 + j12.97} = \frac{1.0\underline{/-1.772^\circ}}{18.67\underline{/44.0^\circ}} = 0.05356\underline{/-45.77^\circ} \quad \text{per unit}$$

$$I_2 = \frac{V_2}{jX_{eq} + Z_{L2} + Z_{load2}} = \frac{0.03327\underline{/216.59°}}{18.67\underline{/44.0°}}$$

$$= 0.001782\underline{/172.59°} \quad \text{per unit}$$

The phase a source current is then, using (8.1.20),

$$I_a = I_0 + I_1 + I_2$$

$$= 0 + 0.05356\underline{/-45.77°} + 0.001782\underline{/172.59°}$$

$$= 0.03511 - j0.03764 = 0.05216\underline{/-46.19°} \quad \text{per unit}$$

Using $I_{baseH} = \frac{75{,}000}{480\sqrt{3}} = 90.21$ A,

$$I_a = (0.05216)(90.21)\underline{/-46.19°} = 4.705\underline{/-46.19°} \quad \text{A}$$

■

8.8 Power In Sequence Networks:

* The total complex power delivered to three-phase load:

$$S_p = V_{ag}I_a^* + V_{bg}I_b^* + V_{cg}I_c^* = [V_{ag}\, V_{bg}\, V_{cg}] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = \boldsymbol{V}_p^{\mathrm{T}}\boldsymbol{I}_p^*$$

$$= 3\boldsymbol{V}_s^{\mathrm{T}}\boldsymbol{I}_s^* = 3[V_0 + V_1 + V_2] \begin{bmatrix} I_0^* \\ I_1^* \\ I_2^* \end{bmatrix} = 3(V_0I_0^* + V_1I_1^* + V_2I_2^*) = 3S_s$$

EXAMPLE 8.9 Power in sequence networks

Calculate S_p and S_s delivered by the three-phase source in Example 8.6. Verify that $S_p = 3S_s$.

SOLUTION Using (8.5.1),

$$S_p = (277\underline{/0°})(25.15\underline{/+46.76°}) + (260\underline{/-120°})(25.71\underline{/-196.34°})$$

$$+ (295\underline{/115°})(26.62\underline{/-73.77°})$$

$$= 6967\underline{/46.76°} + 6685\underline{/43.66°} + 7853\underline{/41.23°}$$

$$= 15{,}520 + j14{,}870 = 21{,}490\underline{/43.78°} \quad \text{VA}$$

In the sequence domain,

$$S_s = V_0I_0^* + V_1I_1^* + V_2I_2^*$$

$$= 0 + (277.1\underline{/-1.77°})(25.82\underline{/45.55°})$$

$$+ (9.218\underline{/216.59°})(0.8591\underline{/-172.81°})$$

$$= 7155\underline{/43.78°} + 7.919\underline{/43.78°}$$

$$= 5172 + j4958 = 7163\underline{/43.78°} \quad \text{VA}$$

Also,

$$3S_s = 3(7163\underline{/43.78°}) = 21{,}490\underline{/43.78°} = S_p$$

■